Ponderomotive acceleration of electrons by a tightly focused intense laser beam

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Ponderomotive force driven acceleration of an electron at the focus of a high-intensity short-pulse laser is considered. Accounting for the asymmetry of acceleration and deceleration due to the evolution of the Gaussian laser beam waist, the energized electron is extracted from the laser pulse by the longitudinal ponderomotive force. It is shown that an electron's energy gain in the range of MeV can be realized for laser intensities above 10^{19} W μm^2/cm^2. Final energy gain as a function of the scattering angle and the electron's initial position has also been discussed.

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I. INTRODUCTION

The new technique of chirped pulse amplification (CPA) [1] allows one to generate laser pulse with focused intensities far above 10^{18} W μm^2/cm^2. Intense focused optical beams can reach field levels greater than 10^{12} V/m in vacuum, which is many orders of magnitude greater than that produced by the conventional accelerator. Such enormous fields have stimulated a great deal of research in laser-driven acceleration concepts in vacuum [2–10] and plasma [11–20]. Electrons accelerated to relativistic energies have been observed in experiments [12,13,21].

It is well known that planar electromagnetic waves do not serve the purpose of electron acceleration. This is true even when the light pressure effects are included, since when a wave overtakes an electron, the radiation pressure pushes the electron forward in the ascending (leading) front and backward in the descending (trailing) part of the laser pulse. As a result, the electron does not acquire net acceleration. However, if after being accelerated the electron leaves the interaction region before being decelerated, it will have gained the energy. So the extraction of the energized electron from the laser pulse is a crucial issue for laser acceleration of electrons. Hartemann *et al.* [4] proposed an extraction mechanism known as ponderomotive scattering. They think that when the quiver amplitude of the electron driven by the laser field exceeds the focal spot radius of a Gaussian beam, the restoring force acting on the charge decays exponentially, and the electrons are scattered away from the focus. However the estimated theoretical threshold laser intensity needed for this mechanism to operate is rather larger than that shown in experiments and simulations. In fact, in addition to the transverse quiver motion, an electron is driven forward by the longitudinal ponderomotive force (light pressure) in the ascending front of the pulse. In the relativistic regime, this slowly varying drift motion can be much larger than the quiver motion. Using a model that accounts for electron's drift motion but neglects its fast-varying quiver motion, Yu *et al.* [6] proposed that the electron is mainly accelerated by the longitudinal ponderomotive force and extracted by the radial ponderomotive force of the laser pulse. This theory can explain the simulations and the experiments well. Since the Rayleigh length for a wide enough laser beam waist is much longer than the electron's drifting distance in the electromagnetic field, in those cases the change of the laser beam waist's width can be neglected.

However, now the laser beam can be focused down to few microns, such as 5λ (λ is the wavelength and $\lambda=1$ μm in our calculations), or even to λ [22]. In this case, the corresponding Rayleigh length is comparable to or even shorter than the laser-electron interaction drift distance. Therefore, the change of the laser beam waist cannot be neglected anymore. In this paper, we extend the work of Yu *et al.* [6] by considering the change of the laser beam waist and finding that the longitudinal ponderomotive force can not only accelerate the electron but also extract the energized electron from the laser pulse. When the laser pulse overlaps a stationary electron in the vicinity of the focus, the ponderomotive force is intense and the electron is accelerated effectively because the laser beam waist is narrow and the intensity is high. When the laser pulse overtakes and begins to decelerate the electron, both the electron and the laser pulse are far from the focus where the laser beam waist is wide but the intensity is weak, so the ponderomotive force is weak and the deceleration is ineffective. Therefore the acceleration and deceleration will not be counteracted, and we found that the energy gain for electrons in the range of MeV can be realized for a laser intensity above 10^{19} W μm^2/cm^2. The dependence of the energy gain on the electron's initial position for this acceleration mechanism is presented in this paper, and the relationship between the energy gain and the scattering angle is discussed.

II. THE ACCELERATED MODEL AND FORMULATION

For a circular polarized tightly focused laser pulse, the vector potential can be expressed as [23]

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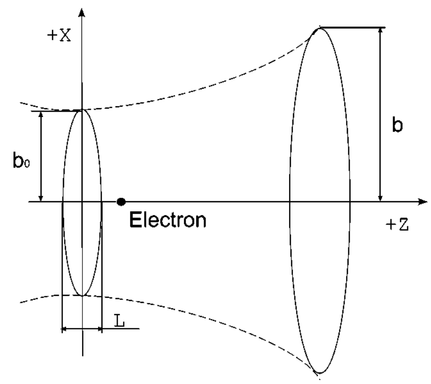

FIG. 1. Schematic geometry of electron acceleration by a circular polarized Gaussian laser beam, we assume that the laser pulse propagates along $+z$ axis.

$$\mathbf{a}=a_0\exp(-\eta^2/L^2-\rho^2/b^2)(b_0/b)\hat{\mathbf{a}}, \tag{1}$$

where $\hat{\mathbf{a}}=\cos(\phi)\hat{\mathbf{x}}+\sin(\phi)\hat{\mathbf{y}}$, $\rho^2=x^2+y^2$, L and b are the pulse width and beam waist, a_0 is the peak amplitude normalized by mc^2/e, and $b=b_0(1+z^2/z_f^2)^{1/2}$, where b_0 is the minimum spot size and $z_f=b_0^2/2$ is the corresponding Rayleigh length. $\phi=\phi_p-\phi_G-\phi_0+\phi_R$, where $\phi_P=z-t=\eta$, $\phi_G=z/z_f$, $\phi_R=(x^2+y^2)/[2R(z)]$, and $R(z)=z(1+z_f^2/z^2)$. Note that ϕ_0 is a constant, ϕ_P is the plane-wave phase, ϕ_G is the Guoy phase associated with the fact that a Gaussian beam undergoes a total phase change of π as z changes from $-\infty$ to $+\infty$, ϕ_R is the phase associated with the curvature of the wave fronts, and that $R(z)$ is the radius of a curvature of a wave front intersecting the beam axis at the coordinate z. In the above definitions, space and time coordinates are normalized by k_0^{-1} and ω_0^{-1}, respectively, and ω_0 and k_0 are the laser frequency and wave number, respectively. m and e are the electron's mass and charge, respectively. Obviously, the intensity of the laser beam is sensitive to b. At the focus, the laser beam has the highest intensity due to the minimum spot size. The peak intensity at a distance of Rayleigh length is 50% of that in the focus.

In the Cartesian coordinate, the components of the vector potential can be written as

$$a_x=a_L\cos(\phi),\quad a_y=a_L\sin(\phi), \tag{2}$$

where $a_L=a_0\exp(-\eta^2/L^2-\rho^2/b^2)(b_0/b)$. For satisfying the Coulomb gauge $\nabla\cdot\mathbf{a}=0$, the vector potential exists the longitudinal component, i.e.,

$$a_z=a_L\left[-\frac{2x}{b_0b}\sin(\phi+\theta)+\frac{2y}{b_0b}\cos(\phi+\theta)\right], \tag{3}$$

where $\theta=\pi-\tan^{-1}(z/z_f)$. In fact, a_z is one order of magnitude less than a_x and a_y when the beam width is larger than five times the wavelength.

The configuration of laser-electron interaction is shown in Fig. 1. We assume that the laser pulse propagates along the $+z$ axis and an electron initially stationary is on the axis and near the focus. When the laser beam overlaps the electron, the radiation pressure pushes the electron forward in the ascending front and the electron is accelerated rapidly. In this region, the laser beam waist is near b_0 and the intensity is very high. When the descending part of the pulse begins to decelerate the electron, both the electron and the laser beam are far from the focus and the intensity of laser beam decreases rapidly with the increase of the beam waist. Because of the asymmetry in acceleration and deceleration, the electron can obtain net energy from the pulse when the electron departs from the laser pulse.

The motion of an electron in an electromagnetic wave is described by the Lorentz equation [24]

$$d_t(\mathbf{p}-\mathbf{a})=-\boldsymbol{\nabla}_a(\mathbf{u}\cdot\mathbf{a}), \tag{4}$$

together with an energy equation

$$d_t\gamma=\mathbf{u}\cdot\partial_t\mathbf{a}, \tag{5}$$

where $\mathbf{u}$ is the velocity of electron normalized by c, $\mathbf{p}=\gamma\mathbf{u}$ is the normalized momentum, $\gamma=(1-\mathbf{u}^2)^{-1/2}$ is the relativistic factor or normalized energy, and $\boldsymbol{\nabla}_a$ in Eq. (4) acts on $\mathbf{a}$ only. Note that Eqs. (4) and (5) are exact.

Substituting Eqs. (2) and (3) in Eqs. (4) and (5), and after some straightforward algebra, we find

$$\gamma d_tu_x=(1-u_x^2)\partial_ta_x+u_y(\partial_ya_x-\partial_xa_y)+u_z(\partial_za_x-\partial_xa_z)-u_xu_y\partial_ta_y-u_xu_z\partial_ta_z, \tag{6}$$

$$\gamma d_tu_y=(1-u_y^2)\partial_ta_y+u_x(\partial_xa_y-\partial_ya_x)+u_z(\partial_za_y-\partial_ya_z)-u_xu_y\partial_ta_x-u_yu_z\partial_ta_z, \tag{7}$$

$$\gamma d_tu_z=(1-u_z^2)\partial_ta_z+u_x(\partial_xa_z-\partial_za_x)+u_y(\partial_ya_z-\partial_za_y)-u_xu_z\partial_ta_x-u_yu_z\partial_ta_y, \tag{8}$$

$$d_t\gamma=u_x\partial_ta_x+u_y\partial_ta_y+u_z\partial_ta_z, \tag{9}$$

where u_x, u_y, and u_z are the components of the electron's velocity in x, y, and z directions, respectively. Solving Eqs. (6)–(9), we can obtain electron's trajectory and energy during the interaction.

III. RESULT AND DISCUSSION

For an electron initially at rest, the drifting distance s is $0.63a_0^2L$, which is obtained from Eqs. (4) and (5) by simplifying the electromagnetic field as a planar wave with pulse width L and peak strength parameter a_0. Here, we also regard the electron's drifting distance in focused laser beam as s. If the Rayleigh length is much longer than s, the change of the laser beam waist can be neglected. For example, the drifting distance is about $160\lambda_0$ (λ_0 is the wavelength normalized by k_0^{-1} and can be replaced by 2π) when $a_0=5$,$L=10\lambda_0(33\text{ fs})$. If the beam waist is up to $20\lambda_0$, the corresponding Rayleigh length will be $1257\lambda_0$. In this case, we can neglect the change of the beam waist. Figure 2 shows the electron's trajectory and energy as a function of time t for the above situation. The laser pulse propagates along the $+z$ axis and the electron is at $(0,0,0)$ initially, where the radial

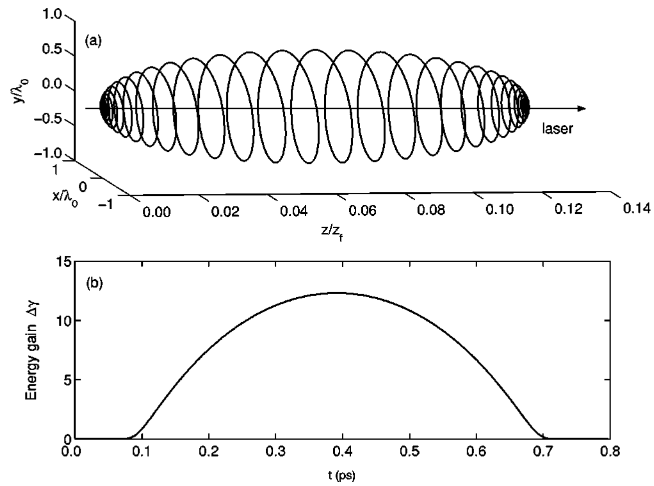

FIG. 2. Electron trajectory in a circular polarized Gaussian pulse electromagnetic wave (a) and electron's energy during the interaction (b) for $a_0 = 5$, $L = 10\lambda_0$, and $b_0 = 20\lambda_0$.

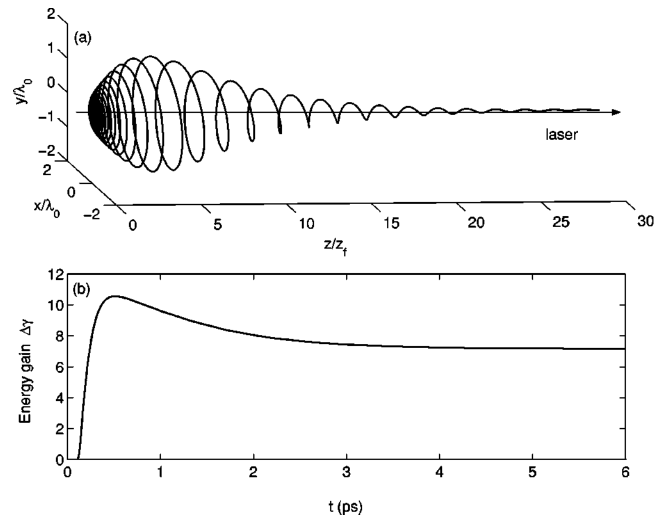

FIG. 3. Electron trajectory in a circular polarized Gaussian pulse electromagnetic wave (a) and electron's energy during the interaction (b) for $a_0 = 5$, $L = 10\lambda_0$, and $b_0 = 5\lambda_0$.

ponderomotive force is about zero. One can clearly see from Fig. 2(a) that the electron's trajectory is screwy. The radial maximum displacement is less than λ_0 and the electron drifts about 10% of the Rayleigh length along the $+z$ axis during the interaction. It is shown in Fig. 2(b) that the electron is accelerated during 0.1–0.4 ps and decelerated in 0.4–0.7 ps. The maximum energy gain $\Delta\gamma$ is more than 12 during the interaction, but the electron's velocity reverts to zero after the interaction. Because the drifting distance along the axis and the maximum radius are much shorter than the Rayleigh length and the beam waist b_0, respectively, the electromagnetic wave acting on the electron is near to a planar wave during the interaction and the vector potential can be rewritten nearly as $\mathbf{a} = a_0 \exp(-\eta^2/L^2)\hat{\mathbf{a}}$. So no energy gain that takes place is reasonable. If the electron is not located on the propagation axis initially, however, the electron can be pushed away from the axis by the radial ponderomotive force and then be extracted from the laser pulse, as described in reference Ref. [6].

If the beam waist is sometimes wide of the wavelength, such as $5\lambda_0$ (the corresponding Rayleigh length is $78\lambda_0$), the interaction scenario is obviously different from the above situation. When $a_0 = 5$, and $L = 10\lambda_0$, the estimated drifting distance along the $+z$ axis is near $160\lambda_0$ and is twice the Rayleigh length, the change of the beam waist cannot be neglected anymore. The calculated electron's orbit and energy gain for an electron initially at (0,0,0) are shown in Figs. 3(a) and 3(b), respectively. At the beginning of the interaction, the electron's orbit is helix and its velocity and energy increase rapidly in the leading front of the pulse. The electron is pushed forward from the focus by the radiation pressure and the laser intensity becomes weaker and weaker with the increase of the laser beam waist. As the pulse is far from the focus, the radial field on propagation axis is near to planar electromagnetic field. As a result, the radial ponderomotive force is so weak that the electron is confined near the axis. When the ascending part of the pulse overtakes the electron, both the electron and the laser pulse are far from the focus and the intensity of the laser pulse is only 30% of that in the focus. So the deceleration by the descending part of the pulse is ineffective and the electron propagates with the laser pulse along the $+z$ axis with a speed very near c. After 5 ps, the electron's displacement along the $+z$ axis is up to 20 times that of the Rayleigh length and the energy gain $\Delta\gamma$ is about 7.2, but the laser intensity is only 0.4% of that in the focus. One can also see from Fig. 3(a) that the final velocity of the electron is nearly parallel with the propagation axis.

If the electron is put on different positions in the focus plane, the energy gain will also be different. Figure 4 shows the dependence of the electron's energy gain on its initial radial position in the focus plane, where $a_0 = 5$ and $b_0 = 5\lambda_0$ (solid line), $a_0 = 3$ and $b_0 = 20\lambda_0$ (dash-dotted line), the pulse width $L = 10\lambda_0$. One can see that the optimal initial radial position where an electron gains maximum energy ap-

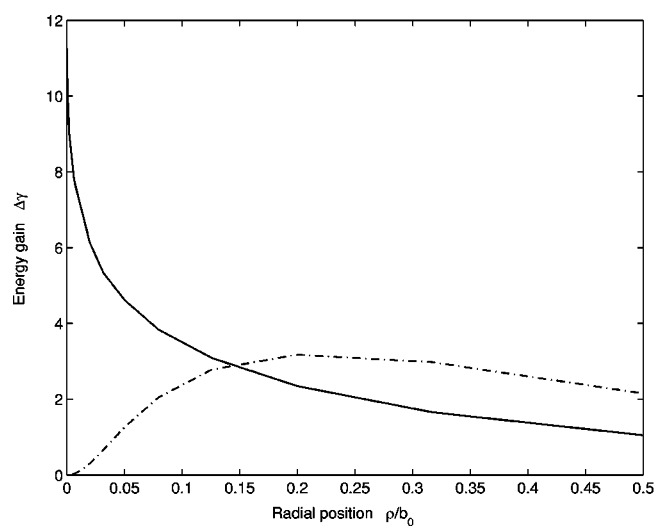

FIG. 4. Dependence of electron's energy gain on its initial radial position. The dash-dotted line and the solid line describe the situations for $a_0 = 3$, $b_0 = 20\lambda_0$ and $a_0 = 5$, $b_0 = 5\lambda_0$, respectively. Some other parameters are $L = 10\lambda_0$.

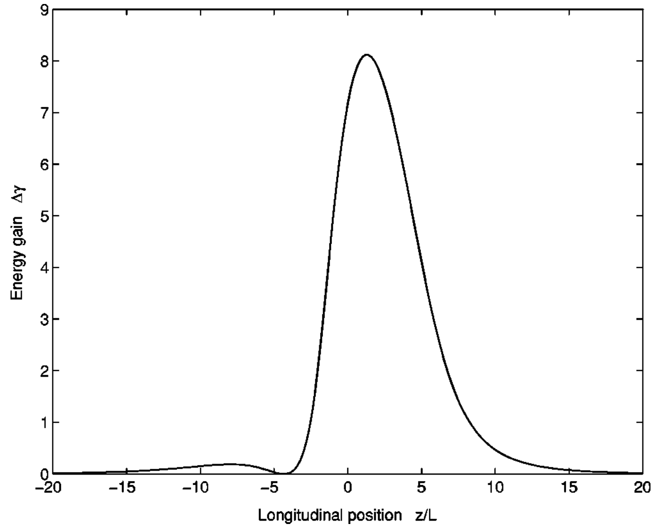

FIG. 5. Dependence of electron's energy gain on its initial longitudinal position. The laser parameters are the same as those in Fig. 3.

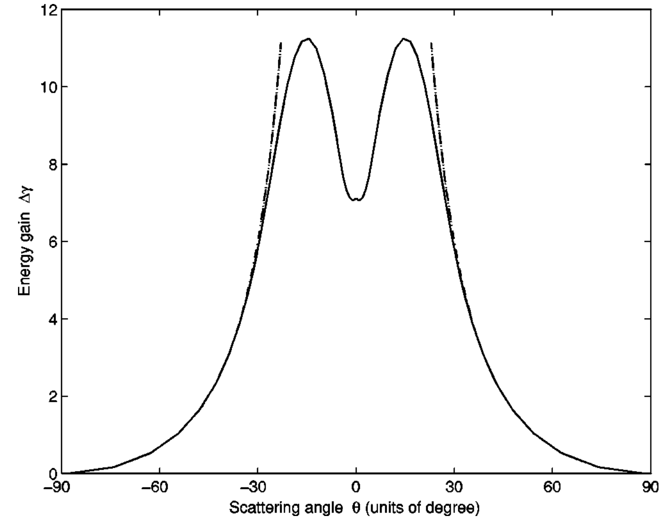

FIG. 6. Dependence of the scattering angle θ on energy gain $\Delta\gamma$. The laser parameters are the same as those in Fig. 3.

pears at $\rho/b_0=0.2$ for the case with wider beam waist, and the corresponding maximum energy gain is about 3.2. We should emphasize that there is no energy gain for the electron initially on the propagation axis when $b_0=20\lambda_0$ and this accords with Fig. 2. On the contrary, the electron initially near the axis obtains maximum energy gain when $b_0=5\lambda_0$, the corresponding maximum energy gain $\Delta\gamma$ is about 11.3. The energy gain decreases rapidly with the increase of ρ/b_0 and electrons distributed in the region of $\rho/b_0>0.1$ cannot be accelerated effectively. Comparing the case for $a_0=5$ and $b_0=5\lambda_0$ with that for $a_0=3$ and $b_0=20\lambda_0$, one can clearly see that the maximum energy gain for $a_0=5$ and $b_0=5\lambda_0$ is larger than that for $a_0=3$ and $b_0=20\lambda_0$, though the power of the laser pulse for $a_0=5$ and $b_0=5\lambda_0$ is an order of magnitude less than that for $a_0=3$ and $b_0=20\lambda_0$. So we may conclude that the electron acceleration is more effective by more tightly focused laser pulse. The narrowness of the peak in solid line implies that only a small fraction of electrons is accelerated to the highest energies. But the electron is not always at rest initially in fact. If the electron has a small energy of 40 eV with the initial velocity along the $+x$ axis, the energy of the accelerated electron is about $\gamma=10$ after the interaction when the electron is put on the point $(-b_0/10,0,0)$ initially. So the electron beam can be accelerated well by a tightly focused laser beam.

Figure 5 shows the dependence of energy gain on its initial longitudinal position. In the calculation, the electron is put on the propagation axis. The laser parameters are the same as those in Fig. 3. One can see from Fig. 5 that there is an optimal initial position where an electron gains maximum energy. Concretely, the initially stationary electron at $z/L=1$ gains the biggest energy, which is up to 8.1. If the electron is put on the propagation axis but far away from the focus, there is nearly no energy gain after the interaction. This can be explained by the fact that both the acceleration and the deceleration are ineffective because of the weak pulse intensity in the region far from the focus. For example, the intensity at $z/L=20$ is only 13% of that in the focus due to the wider beam waist, and the corresponding strength parameter a is less than 1. It is interesting that there is a low apex at $z/L=-7.8$ shown in Fig. 5, where the electron obtains the extremal energy gain. In fact, the electron around this position initially is accelerated ineffectively but decelerated effectively. In other words, the electron is accelerated weakly because of the weak laser intensity. During the acceleration period, as the electron and the laser pulse get close to the focus, the intensity of the laser pulse becomes higher and higher. When the ascending part of the pulse overtakes and decelerates the electron, the laser intensity is so high that it not only makes the electron's velocity along the $+z$ axis zero but also lets it obtain the velocity along the $-z$ axis. Since the electrons put on different positions on the axis can obtain the velocity along $+z$ axis and $-z$ axis, there must exist a position where the electron has no energy gain because the acceleration and deceleration can be counteracted by each other. We can easily find that position from Fig. 5, i.e., $z/L=-4.4$.

In general, the velocity of accelerated electron is not always parallel to the $+z$ axis. If electrons are distributed in different positions initially in the focus plane, they will escape from the laser pulse with different scattering angles and different energy gains. When neglecting the change of the beam waist, there is a remarkable relation between the energy gain $\Delta\gamma$ of the electron and the scattering angle θ [5], i.e.,

$$\theta=\arctan(\sqrt{2/\Delta\gamma}). \tag{10}$$

In this case, the scattering angle cannot be zero because the energy gain of accelerated electron is finite. But if considering the change of the laser beam waist, the dependence of scattering angle on its energy gain is different from Eq. (10). Figure 6 shows the relationship of the scattering angle θ and the energy gain $\Delta\gamma$. The solid line and the dash-dotted line describe the situations considering and neglecting the evolution of the beam waist, respectively. The laser parameters in

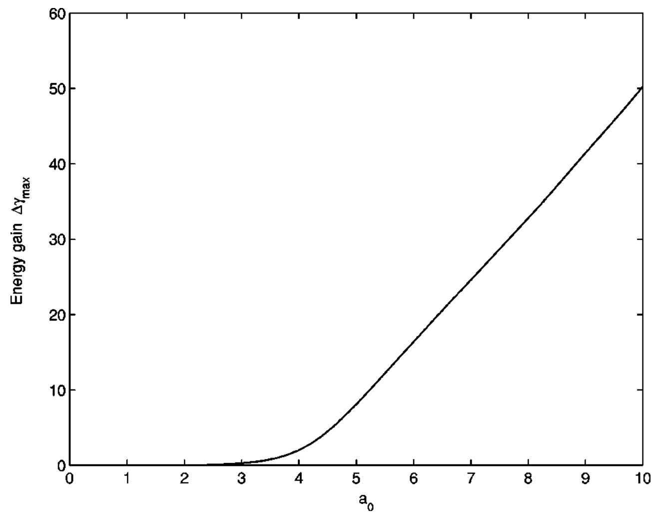

FIG. 7. Dependence of the maximum energy gain $\Delta\gamma_{max}$ on the light amplitude a_0. The laser pulse width $L=10\lambda_0$, the minimum spot size $b_0=5\lambda_0$.

this calculation are the same as those in Fig. 3. When the electron is away from the origin in the focus plane, there is no difference for the relationships between scattering angle and energy gain whether considering the evolution of the beam waist. One can also clearly see that there is no electron in the small scattering angle region if neglecting the evolution of the beam waist. While considering the evolution of the beam waist, we find the scattering angles of energetic electrons reduced obviously. The maximum energized electron escapes from the laser pulse with a 15° degree scattering angle. Note that the energy gain is 7.2 when the scattering angle is zero, which reminds us that the collimated MeV electrons could be generated in the forward direction by a tightly focused laser.

Figure 7 shows the electron's maximum energy gain as a function of the laser amplitude. When $a_0<2$, the approximately drifting distance is shorter than the Rayleigh length, so the evolution of the beam waist can be neglected, and there is nearly no net energy gain. The energy gain is proportion to a_0^2 as $2<a_0<5$. When $a_0>5$, the energy gain increases with a_0 linearly. When $a_0=10$, the energy gain $\Delta\gamma=50$.

IV. CONCLUSION

We have described a mechanism of extracting energetic electrons from a tightly focused laser beam. The initially stationary electron can be accelerated effectively in the vicinity of the focus where the laser beam waist is narrow and the intensity is high, but decelerated ineffectively far from the focus where the laser beam waist is wide and the intensity is weak. Because of the asymmetry of the laser intensity in acceleration and deceleration, the electron can obtain net energy gain from the laser pulse when the electron departs from the laser pulse. When considering the change of the laser beam waist, Eq. (10) is not exact to describe the dependence of the scattering angle on the energy gain. The scattering angles for electrons initially on the propagation axis are near to zero. Comparing the mechanism of electron acceleration in this paper with the method described in Ref. [6], we find that tightly focused laser pulse has an advantage of generating collimated MeV electron beam in the forward direction.

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